SELF-EXPRESSIVE NUMBER NAMES

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If one sets A=1, B=2, ... Z=26, it is well-known that no number-name is self-expressive (15+14+5 does not add to ONE, 20+23+15 to TWO, etc.). However, there are other ways to arithmetically combine numbers. If subtraction as well as addition is allowed, then five of the first twenty number-names can express themselves:

SEVEN -19+5+22-5+14 = 36-29 = 7
THIRTEEN -20+8+9-18+20-5+5+14 = 56-43 = 13
FOURTEEN +6+15-21+18+20-5-5-14 = 59-45 = 14
FIFTEEN -6-9+6+20-5-5+14 = 40-25 = 15
SEVENTEEN -19-5-22+5+14+20+5+5+14 = 63-46 = 17

One can similarly self-express half of the number-names between TWENTY and EIGHTY: 21-3-5-8-9, 30-2-4-6-7, 40-2-4-6-7, 50-2-4-6-7, 61-3-5-8-9. 70-2-4-6-7. However, EIGHTY cannot be self-expressed because its letters sum to only 74, and NINETY similarly fails because it sums to 87. NINETY-FIVE, despite having ten numbers to work with, fails just as ONE through SIX did; one cannot allocate plus and minus signs to 14,9,14,5,20,25,6,9,22,5 so that positive numbers add to 112 and negative ones to 17.

As number-names grow larger, self-expressive ones become scarcer for both of the above reasons. The largest self-expressive ones appear to be

TWO HUNDRED NINETY-SIX (58+109+87+52 = 306) 301-5(E) = 296
THREE HUNDRED TWENTY-TWO (56+109+107+58 = 330) 326-4(D) = 322
THREE HUNDRED TWENTY-FOUR (56+109+107+60 = 332) 328-4(D) = 324

One can generate additional self-expressive number-names by allowing multiplication and division. One can produce even more by allowing the square root and the factorial (for example, 4! = 4x3x2x1 = 24). However, the components must be used in order, a requirement that is irrelevant when only addition and subtraction is considered. Here are additional self-expressive numbers less than or equal to TWENTY:

THREE (20,8,18,5,5) $20/(-8+18) + 5/5 = 3$
EIGHT (5,9,7,8,20) $5\sqrt{(9+7)} + 8 - 20 = 8$
TEN (20,5,14) $-20/5 + 14 = 10$
ELEVEN (5,12,5,23,5,14) $-5 - 12/5 + 22/5 + 14 = 11$
TWELVE (20,23,5,12,22,5) $-20 - 23 + 5 + (12-22)(-5) = 12$
SIXTEEN (19,9,24,20,5,5,14) $19 + 9 - 24 - 20/(5+5) + 14 = 16$
EIGHTEEN (5,9,7,8,20,5,5,14) $(5+9)/(-7) + 8 - 20/(5+5) + 14 = 18$
NINETEEN (14,9,14,5,20,5,5,14) $14 + 9 - 14 - 5 - (20/5) + 5 + 14 = 19$
TWENTY (20,23,5,14,20,25) $-(-20+23)! - 5 - 14 + 20 + 25 = 20$

Alas, most three-letter and four-letter number-names possess too few components to work with.

The largest self-expressive number-name using all the arithmetic operations (including factorial) is extraordinarily difficult to determine, requiring the use of a computer. Not counting the six single-digit number-names having only three or four letters, it should be somewhat easier to determine the smallest number-name that is *not* self-expressive.

I am indebted to Dave Morice for the concept of self-expressive numbers by arithmetic operation.

CARTOON

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